**Completing the Square (for Lines)**

Ex:

1) Move the constant to the other side.

2) Take the b term, divide it by 2, and square. (y = ax2 + bx + c)

b = -1

= = 4

3) Add your to both sides of the equation.

y+ 21 + 4 = (x2 - 4x + (4))

4) Combine the constants (if the sum is a fraction, it’s best to write as an improper fraction).

y + 25 = (x2 **-** 4x + (4))

5) Now, write the new form of the equation as (x <the sign of b> <the square root of c>)2.

y + 25 = (x - 2)2

6) Move the constant on the y side to the other side.

y = (x - 2)2 - 25

\*What does this mean? This form is called the *vertex form*.

The VERTEX FORM is written as y = a(x - h)2 + k, where the vertex is (h, k).

Question: What is the vertex of the equation above?

Answer: The vertex is at (2, -25), but notice how the sign for the x-coordinate is not -2! You must take **the opposite sign** for the x-coordinate, or h value, when writing the coordinates of the vertex from the vertex form.

**Completing the Square (for Circles)**

EQUATION OF A CIRCLE: (x - h)2 + (y - k)2 = r2, where (h, k) = center of circle & r = radius

Ex: x2 - 4x + y2 - 6y - 3 = 0

1) Move the constant to the other side of the equation and group the x and y variables.

(x2 - 4x) + (y2 - 6y) = 3

2) Complete the square like the steps above! (Be careful with the numbers and signs! They can get a bit messy!)

(x2 - 4x + (4)) + (y2 - 6y + (9)) = 3 + 4 + 9

3) Change the form:

(x - 2)2 + (y - 3)2 = 16

Answer: The equation of the circle is (x - 2)2 + (y - 3)2 = 16.

→ BUT WAIT! What is the center and radius?

The center of the circle is (h, k) in the equation (x - h)2 + (y - k)2 = r2, which means you **have to flip the signs**. If the equation is (x - 2)2 + (y - 3)2 = 16, then the center must be C(2, 3).

In the equation (x - h)2 + (y - k)2 = r2, radius is *squared*. Therefore, to find the radius of the circle, we must find the **square root of r2**. If the equation is (x - 2)2 + (y - 3)2 = 16, then the radius is = 4.

| FINAL ANSWER:  (x - 2)2 + (y - 3)2 = 16  C(2, 3); r = 4 units |
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